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Influence of Nonconservative Differencing on Transonic Streamline Shapes

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THE basic relaxation technique introduced by Murman and Cole[1] has been used extensively in computing transonic flowfields. A deficiency was later recognized in the original finite-difference scheme and Murman[2] introduced a conservative finite-difference scheme, which included a shock-point operator, in order to correct it. Nevertheless, the original nonconservative scheme continues to be used by most investigators since it seems to give shock jumps and locations at the surface of the configuration more nearly like those observed in experiments. The purpose of this Note is to demonstrate that these nonconservative shocks, which may extend well into the flowfield, destroy the global mass balance by producing mass at the shock. In a transonic internal (or confined) flow this lack of mass balance may prove to be more crucial than is the case for an unconfined external flow.

The present results and observations were prompted by streamtube anomalies first noted in calculations pertaining to wind tunnel flows in both 2 and 3 dimensions. The influence of conservative vs nonconservative finite-difference formulation on the global mass balance is readily observed by computing streamline shapes in an external 2-D transonic flow. This Note presents such samples; some additional detail and material was given limited distribution in Ref. 3.

Problem Description

A NASA Langley study of one concept for minimizing wind-tunnel interference involves contouring the upper and lower (initially solid) walls of a small 2-D facility according to numerical results obtained from nonlinear flow solutions. Calculated free-air streamline deflections at the proposed tunnel-wall locations for subsonic lifting flow showed that a streamtube, roughly the dimensions of the tunnel, returned to its far-upstream size far downstream of the model. That is, there was a global mass balance in the calculation. Similar calculations for supercritical lifting flow with shocks, however, showed that the size of the streamtube had increased from far upstream to far downstream, indicating that mass had been introduced. These results were generated using a program which employed the Garabedian and Korn[4] nonconservative finite-difference scheme.

A computer program recently developed by South and Brandt[5] contained the Murman[2] conservative finite-difference scheme and was easily modified to use the Garabedian and Korn[4] nonconservative finite-difference scheme. This program solves the transonic small disturbance equation for only symmetric flow, but incorporates several iterative solution techniques. For the results presented here, the equally-spaced computational grid was analytically stretched so that the physical grid extended to infinity in both the streamwise and normal directions. Streamline shapes were obtained along several grid lines by a streamwise integration of the normal component of the perturbation velocity.

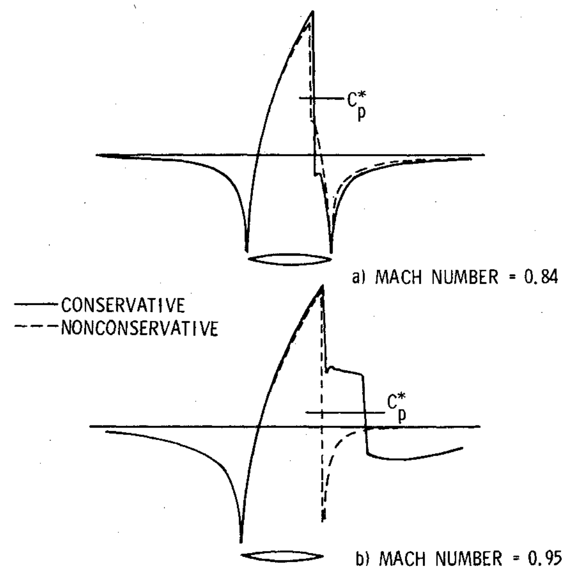

Fig. 1 Computed pressure coefficients along symmetry line for nonlifting transonic flow past 10% thick parabolic arc airfoil.

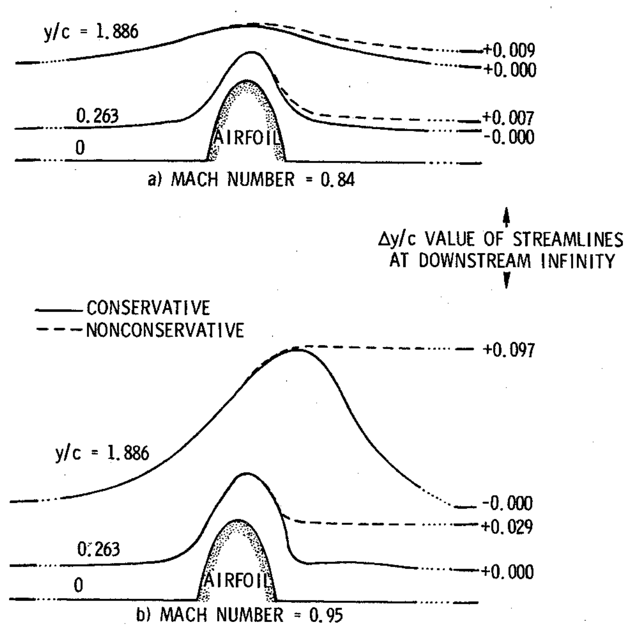

Fig. 2 Computed streamline deflections for nonlifting transonic flow past 10% thick parabolic arc airfoil.

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Comparisons of conservative and nonconservative results were made with all aspects of the computations identical except the finite-differencing scheme. In all cases the calculations were iterated until the root-mean-square value of the true residual was of the order of the truncation error. Although the solution itself was observed to change somewhat for a given case when the analytical stretching was changed, it is felt that the relative streamline shape effects presented here indicate that a nonconservative finite-difference scheme destroys the global mass balance in a supercritical flow calculation when shocks are present.

Results

Comparison cases were run for a 10% thick parabolic arc airfoil at zero incidence for freestream Mach numbers of 0., 0.70, 0.84, and 0.95. These represent incompressible, subcritical, mildly supercritical, and strongly supercritical flow conditions, respectively. The computational grid was 128 streamwise by 33 in the normal direction (the physical half-space, airfoil mean plane to infinity). Streamline deflections were computed along several grid lines for all cases; only three are shown in the figures.

The incompressible ($M=0$) and subcritical ($M=0.70$) results for pressure distributions and streamline deflections were identical for conservative and nonconservative finite differencing. In all cases the computed streamtubes returned to their proper size. Since the conservative and nonconservative schemes differ only at points where the flow is supersonic, the results were expected to agree.

Figure 1 shows the streamwise distribution of pressure coefficients along the symmetry line $y=0$ for both the mildly ($M=0.84$) and strongly ($M=0.95$) supercritical flows. Conservative results are given by the solid curves while nonconservative results are given by dashed curves. As others have shown in the past, one effect of the conservative scheme is to locate the shock wave further downstream on the airfoil surface. Figure 1a shows that the mildly supercritical flows are similar. It can be seen in Fig. 1b, however, that the strongly supercritical flows are no longer similar. In the nonconservative case there is a normal shock at the airfoil trailing edge whereas conservative differencing gives a weak oblique shock wave at the trailing edge followed by a normal shock located about 1/2 a chord length downstream of the airfoil.

Computed streamline deflections (from straight lines) $\Delta y/c$ for both mildly and strongly supercritical flows are shown in Fig. 2. Note that the scale of the ordinate ($\Delta y/c$) has been magnified 20 times that of the abcissa (x/c) for clarity. Tick marks at the edges of the figure show asymptotic values. The zero levels of streamline deflection (i.e., $\Delta y/c$ at upstream infinity) for three different streamlines are given by the tick mark at the left edge. The deflection curves are labeled with the upstream infinity value of y/c for the streamline itself. Tick marks at the right edge of the figure denote the levels of streamline deflection $\Delta y/c$ at downstream infinity; numerical values are also indicated. The results for mildly ($M=0.84$) supercritical flow are shown in Fig. 2a. Observe that the streamline deflections at downstream infinity show the conservative streamtube sizes return to their upstream infinity values whereas the nonconservative ones do not. For this case, the shock wave extends about 1/2 a chord length into the flow. Similar results are shown for the strongly ($M=0.95$) supercritical flow in Fig. 2b. Here it is very evident that the nonconservative streamline deflections do not return to zero far downstream of the airfoil. On the other hand, however, the conservative streamtubes are seen to return to their proper size far downstream. For this case, the shock waves extend several chord lengths into the flow.

Use of a nonconservative finite-difference scheme in transonic flow calculations destroys the global mass balance when shocks are present. The mass created by the nonconservative operator at the shock produces a nonphysical swelling of the inviscid streamtubes which persists far downstream. Perhaps the fortuitous agreement between the nonconservative and experimental results comes about because this streamtube swelling effect simulates a viscous wake or thickened boundary layer downstream of a shockwave. In any case, it is felt that the conservative finite-difference scheme should be used in applications where the streamtube effects are important, such as internal or confined flows.

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Technical Comments

Comment on "Rapid Finite-Difference Computation of Subsonic and Slightly Supercritical Aerodynamic Flows"

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At the end of their paper,[1] Martin and Lomax state that they consider their main contribution to be the use of the Aitken/Shanks extrapolation formula

$$u^* = (u_1 u_3 - u_2^2)/(u_1 - 2u_2 + u_3) \qquad (1)$$

to accelerate iterative convergence. This formula is well known to produce very useful results when properly applied; it can also produce indifferent or poor results when used inappropriately. My main purpose in writing this comment is to emphasize the conditions under which this formula can be used to extrapolate to the sum of a series. A secondary purpose is to give some additional discussion of the "one-dimensional example."

What was shown by Aitken, Shanks, and others is that the terms of a convergent power series, and of many divergent series, eventually behave like the terms of a geometric series to within some acceptable accuracy, at which point the extrapolation formula [Eq. (1)] can be applied to sum the tail of the series. This is not quite the same as applying the for-

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